On the topological reconstruction of endomorphism monoids of ω-categorical structures

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joint work with Michael Pinsker, J. de la Nuez Gonzales, and Zaniar Ghadernezhad

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Outline

Topology

$\Omega :=$ a countably infinite set.
Ω^Ω is the **full monoid** on Ω (wrt composition of functions).
Ω^Ω is endowed with the **pointwise convergence topology** τ_{pw}:

$$\text{for } a, b \in \Omega,\ \mathcal{U}_{(a,b)} := \{f \in \Omega^\Omega \mid f(a) = b\},$$

form a sub-basis of open sets for τ_{pw}.

Ω^Ω with τ_{pw} is a **Polish topological semigroup**:

- composition is continuous;
- **Polish**:=separable and completely metrisable.

S_Ω, the group of permutations of Ω, is a Polish topological group.

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Topology and structures

We have the correspondences:

$$\left\{ \begin{array}{c} \text{closed subgroups} \\ \text{of } S_\Omega \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} \text{automorphism groups of} \\ \text{structures with domain } \Omega \end{array} \right\}.$$

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Spaces of symmetries

$\mathbb{A}$:= a structure with domain Ω.
We can associate with $\mathbb{A}$ various spaces of symmetries:

- its **automorphism group** $\mathrm{Aut}(\mathbb{A})$;
- its **monoid of elementary embeddings** $\mathrm{EEmb}(\mathbb{A})$ (maps $\phi : \mathbb{A} \to \mathbb{A}$ preserving arbitrary first-order formulas);
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Two (still imprecise) questions on reconstruction

Question 1 (reconstructing $\mathbb{A}$?)

What information can we recover about the original structure $\mathbb{A}$ from a given space of symmetries (as a topological group/monoid)?

Question 2 (reconstructing topology?)

To what extent does the algebraic structure of a space of symmetries determine its topological structure?

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ω-categorical structures

Definition 1

$G \curvearrowright \Omega$ is **oligomorphic** if $G \curvearrowright \Omega^n$ has finitely many orbits for each $n \in \mathbb{N}$ in its diagonal action $g \circ (a_1, \ldots, a_n) = (ga_1, \ldots, ga_n)$.
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Examples 2

- $(\mathbb{N}, =)$;
- $(\mathbb{Q}, <)$;
- the random graph;
- countably infinite vector spaces over finite fields.

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If $G \curvearrowright \Omega$ is oligomorphic, acl is **locally finite**:
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G has **no algebraicity** if for all $B \subseteq \Omega$ finite $\mathrm{acl}(B) = B$.
$(\mathbb{N}, =), (\mathbb{Q}, <)$, and the random graph have no algebraicity.
If G has no algebraicity, then $Z(G) = 1$, where

$$Z(G) := \{g \in G | \; \forall h \in G, hg = gh\}\ .$$

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Saturated structures are such that $\mathrm{EEmb}(\mathbb{A}) = \overline{\mathrm{Aut}(\mathbb{A})}$,
where $\overline{G}$ is the closure of G in Ω^Ω (with τ_{pw}).

Topology and bi-interpretations

Definition 2

An **interpretation** of $\mathbb{B}$ in $\mathbb{A}$ is a partial surjection $I : A^d \to B$ for some $d \in \mathbb{N}$ such that for each relation R of $\mathbb{B}$ defined by an atomic formula, $I^{-1}(R)$ is definable in $\mathbb{A}$ (without parameters).

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Asking that $I^{-1}(R)$ is existentially positively definable we get an **existential positive bi-interpretation**.

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Interpretations compose naturally.
If I is an interpretation of $\mathbb{B}$ in $\mathbb{A}$ and J is an interpretation if $\mathbb{A}$ in $\mathbb{B}$, we say that $\mathbb{A}$ and $\mathbb{B}$ are **bi-interpretable** if the induced partial functions $I \circ J$ and $J \circ I$ are definable in $\mathbb{B}$ and $\mathbb{A}$ respectively.

Topology and bi-interpretations

Theorem 2 (Coquand in Ahlbrandt and Ziegler 1986, cf. Lascar 1991)

Let $\mathbb{A}$ and $\mathbb{B}$ be ω-categorical structures. Then, TFAE:

- $\mathbb{A}$ *and* $\mathbb{B}$ *are bi-interpretable;*
- $\mathrm{Aut}(\mathbb{A})$ *and* $\mathrm{Aut}(\mathbb{B})$ *are isomorphic as topological groups;*
- $\mathrm{EEmb}(\mathbb{A})$ *and* $\mathrm{EEmb}(\mathbb{B})$ *are isomorphic as topological semigroups.*

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Theorem 3 (Bodirsky and Junker 2011)

Let $\mathbb{A}$ and $\mathbb{B}$ be ω-categorical.
If $\mathbb{A}$ and $\mathbb{B}$ are existentially positively bi-interpretable, then $\mathrm{End}(\mathbb{A})$ and $\mathrm{End}(\mathbb{B})$ are isomorphic as topological monoids.
The converse holds as long as neither of $\mathbb{A}$ and $\mathbb{B}$ have a constant endomorphism.

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Automatic homeomorphicity

Definition 4 (Automatic homeomorphicity)

Let $\mathcal{S}$ be a topological semigroup.
$\mathcal{S}$ has **automatic homeomorphicity** (AH) if every semigroup isomorphism between $\mathcal{S}$ and a closed submonoid of Ω^Ω is a homeomorphism.

The same definition makes sense for topological groups with respect to closed subgroups of S_Ω.

Automatic homeomorphicity for automorphism groups

There are well-established methods to prove automatic homeomorphicity for $G := \mathrm{Aut}(\mathbb{A})$:

- Show that G has the **small index property** (SIP): every subgroup of index $\leq \aleph_0$ is open.
- Show that G has a **weak ∀∃-interpretation** (a.k.a. Rubin's method).

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Examples 5

- $(\mathbb{N}, =)$ (Dixon, Neumann, and Thomas 1986);
- ω-categorical ω-stable structures, and the random graph (Hodges, Hodkinson, Lascar, and Shelah 1993);
- $(\mathbb{Q}, <)$ and the countable atomless Boolean algebra (Truss 1989).

[1] Some authors ask this for index $< 2^{\aleph_0}$.

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- the random graph and the generic poset (Rubin 1994);
- generic K_n-free graphs, and Cherlin-Hrushovski example without the SIP (Barbina and Macpherson 2007);

Question 3

Can we lift automatic homeomorphicity from G to $\overline{G}$?

Lemma 6 (Bodirsky, Pinsker, and Pongrácz 2017, Lemma 12)

Let G be a closed subgroup of S_Ω with automatic homeomorphicity. Suppose:

$(\star)$ *the only injective $\Phi \in \mathrm{End}(\overline{G})$ that fixes G pointwise is $\mathrm{Id}_{\overline{G}}$.*

Then $\overline{G}$ has automatic homeomorphicity.

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- (Bodirsky, Pinsker, and Pongrácz 2017): for $(\mathbb{N}, =)$ and the random graph;
- (Behrisch, Truss, and Vargas-García 2017): for $(\mathbb{Q}, <)$;
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- (M. and Pinsker 2025): ALWAYS
  (as long as G is the automorphism group of a countable saturated structure)

Proving lifting

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The following can be extracted from (Pech and Pech 2018):

Proposition 7 (M. and Pinsker 2025)

Let $\Phi \in \mathrm{End}(\overline{G})$ *fix* G *pointwise. Then there is a semigroup homomorphism* $\phi : \overline{G} \to Z(G)$ *such that* $\phi(G) = 1$ *and*

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Note that $(\star)$ follows from the above.

Transfer theorem

Theorem 9 (M. and Pinsker 2025)

Let G be the automorphism group of a countable saturated structure. Suppose that G has automatic homeomorphicity (wrt closed subgroups of S_Ω). Then, $\overline{G}$ has automatic homeomorphicity (wrt closed submonoids of Ω^Ω).

Lifting can fail for $\mathrm{End}(\mathbb{A})$

Theorem 10 (M. and Pinsker 2025)

There ω-categorical $\mathbb{A}$ such that $\mathrm{Aut}(\mathbb{A})$ has automatic homeomorphicity, but $\mathrm{End}(\mathbb{A})$ has a discontinuous automorphism.

$\mathbb{A}$ is ω-stable.

Indeed, it is a reduct of the following finitely homogeneous structure:

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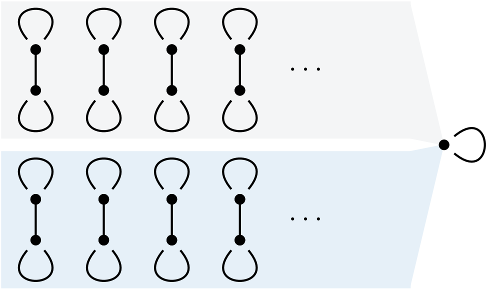

Polish topologies

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For various ω-categorical structures, $\mathrm{End}(\mathbb{A})$ has a unique Polish semigroup topology:

- $(\mathbb{N}, =)$ (Elliott, Jonušas, Mesyan, Mitchell, Morayne, and Peresse 2023);
- the random graph (Elliott, Jonušas, Mitchell, Peresse, and Pinsker 2023);
- $(\mathbb{Q}, \leq)$ (Pinsker and Schindler 2023a).

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However, looking at $\mathrm{EEmb}(\mathbb{A})$, we get:

Proposition 11 (Elliott, Jonušas, Mitchell, Peresse, and Pinsker 2023)

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Minimality and the Zariski topology

What about Polish topologies *coarser* than τ_{pw}?

Often it is possible to show τ_{pw} is the coarsest Hausdorff semigroup topology on $\overline{G}$ by showing it coincides with the **Zariski topology**:

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Analogous statements for groups are usually false:

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Pinsker and Schindler 2023b give an example of a structure for which $\tau_Z \subsetneq \tau_{pw}$ on its endomorphism monoid answering a question of (Elliott, Jonušas, Mesyan, Mitchell, Morayne, and Peresse 2023).

We prove the following more general statement:

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Let $G \curvearrowright \Omega$ have locally finite algebraic closure and non-trivial centre. Then, the semigroup Zariski topology τ_Z on $\overline{G}$ is not Hausdorff.

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Rubin's dream

Definition 14 (Automatic action reconstruction)

Let $\mathbb{A}$ be an ω-categorical structure with no algebraicity.
$\mathrm{Aut}(\mathbb{A})$ has **automatic action reconstruction** (AAR) if whenever $\mathbb{B}$ is another ω-categorical structure with no algebraicity and $\mathrm{Aut}(\mathbb{A}) \cong \mathrm{Aut}(\mathbb{B})$ (as groups), then $\mathbb{A}$ and $\mathbb{B}$ are bi-definable.
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We get the following surprising result:

Theorem 15 (M. and Pinsker 2025)

Let $\mathbb{A}$ be an ω-categorical structure with no algebraicity. Then $\mathrm{EEmb}(\mathbb{A})$ has automatic action reconstruction.

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Theorem 15 (M. and Pinsker 2025)

Let $\mathbb{A}$ be an ω-categorical structure with no algebraicity. Then $\mathrm{EEmb}(\mathbb{A})$ has automatic action reconstruction.

This is just a consequence of

- $\tau_{\mathrm{Z}} = \tau_{\mathrm{pw}}$ (Pinsker and Schindler 2023b);
- bi-interpretation between ω-categorical structures with no algebraicity yield bi-definitions (Rubin 1994; Feller and Pinsker 2025).

Thank you!

A brief recap:

For $\mathbb{A}$ an ω-categorical structure:

- **sometimes we can transfer results from** $\mathrm{Aut}(\mathbb{A})$ **to** $\mathrm{EEmb}(\mathbb{A})$ (automatic homeomorphicity);
- **sometimes** $\mathrm{EEmb}(\mathbb{A})$ **behaves much more wildly than** $\mathrm{Aut}(\mathbb{A})$ (on the number of Polish topologies);
- **sometimes** $\mathrm{EEmb}(\mathbb{A})$ **seems to behave better than** $\mathrm{Aut}(\mathbb{A})$ (minimality of τ_{pw});
- **sometimes hard open problems for** $\mathrm{Aut}(\mathbb{A})$ **have "easy" answers for** $\mathrm{EEmb}(\mathbb{A})$ (automatic action reconstruction);
- $\mathrm{End}(\mathbb{A})$ looks like a **very different beast** from $\mathrm{Aut}(\mathbb{A})$ (which can also be nicer at times).

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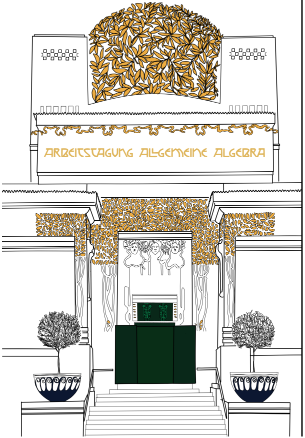

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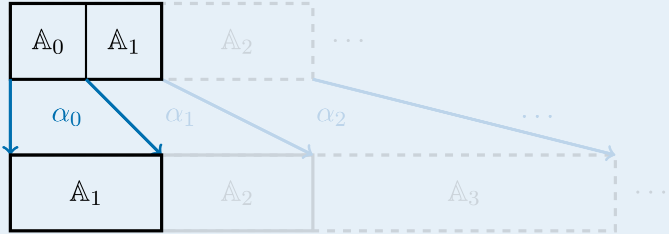
Proof of Lemma.

Let $\mathbb{A}_1$ be saturated with automorphism group G and $\mathbb{A}_0 := h(\mathbb{A}_1)$ with $\alpha_0 := h^{-1}$. We can construct $\mathbb{A}_2 \succeq \mathbb{A}_1$ and an isomorphism $\alpha_1 : \mathbb{A}_1 \to \mathbb{A}_2$ extending α_0. Repeat this process to get an elementary chain $\mathbb{A}_0 \preceq \mathbb{A}_1 \preceq \mathbb{A}_2 \preceq \cdots$ with isomorphisms $\alpha_i : \mathbb{A}_{i+1} \to \mathbb{A}_i$.



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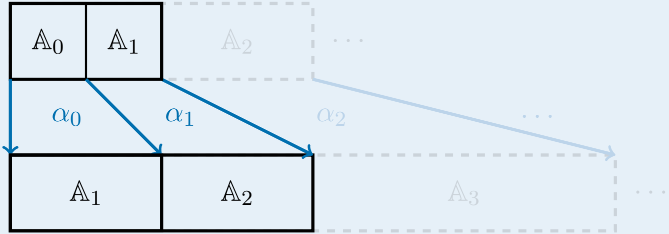

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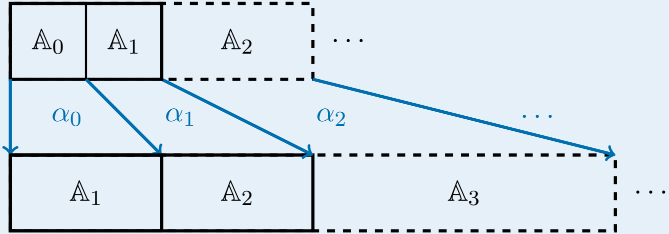

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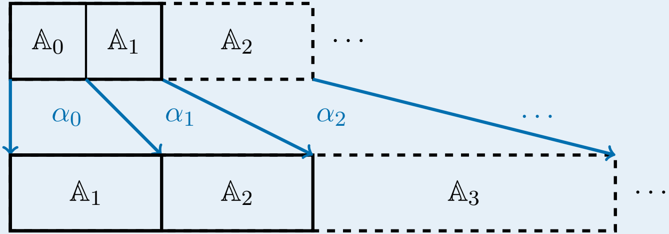

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Take $\mathbb{A} := \bigcup_{i<\omega} \mathbb{A}_i$, $\alpha := \bigcup_{i<\omega} \alpha_i$, and let f be an isomorphism $f : \mathbb{A} \to \mathbb{A}_1$ (exists by saturation). Let $h' := f^{-1}hf \in \mathrm{EEmb}(\mathbb{A})$. We can prove that $(\mathbb{A}_1, h) \cong (\mathbb{A}, h')$.
Finally, $f^{-1}\alpha f h' := \beta \in \mathrm{Aut}(\mathbb{A})$, meaning that

$$\alpha \circ f \circ h' = f \circ \beta,$$

as desired. $\square$

Proving the main Lemma

Proof of Lemma.

Let $\mathbb{A}_1$ be saturated with automorphism group G and $\mathbb{A}_0 := h(\mathbb{A}_1)$ with $\alpha_0 := h^{-1}$. We can construct $\mathbb{A}_2 \succeq \mathbb{A}_1$ and an isomorphism $\alpha_1 : \mathbb{A}_1 \to \mathbb{A}_2$ extending α_0. Repeat this process to get an elementary chain $\mathbb{A}_0 \preceq \mathbb{A}_1 \preceq \mathbb{A}_2 \preceq \cdots$ with isomorphisms $\alpha_i : \mathbb{A}_{i+1} \to \mathbb{A}_i$.

Take $\mathbb{A} := \bigcup_{i<\omega} \mathbb{A}_i$, $\alpha := \bigcup_{i<\omega} \alpha_i$, and let f be an isomorphism $f : \mathbb{A} \to \mathbb{A}_1$ (exists by saturation). Let $h' := f^{-1}hf \in \mathrm{EEmb}(\mathbb{A})$.
We can prove that $(\mathbb{A}_1, h) \cong (\mathbb{A}, h')$.
Finally, $f^{-1}\alpha f h' := \beta \in \mathrm{Aut}(\mathbb{A})$, meaning that

$$\alpha \circ f \circ h' = f \circ \beta,$$

as desired.

□