Extra exercises are marked with a $\star\star$. I DO NOT EXPECT YOU TO ANSWER THEM. I hope they can bring you joy.

EXERCISE 1. Show that the countability of the language in the Omitting Types Theorem is necessary. That is, there is a theory T in an uncountable language with a non-isolated type realised in all models of T.

Definition 1. Let G be a permutation group acting on a countable set B. For each $n \in \mathbb{N}$, G also acts naturally on B^n by

$$g \cdot (b_1, \ldots, b_n) = (g \cdot b_1, \ldots, g \cdot b_n).$$

We say that the action of G on B is **oligomorphic** if for each $n \in \mathbb{N}$ the action of G on B^n has finitely many orbits.

Recall that for M a structure, $\mathrm{Aut}(M)$ denotes the automorphism group of M.

EXERCISE 2. Prove the following are equivalent of a countable complete theory T with infinite models:

- T is ω-categorical;
- for all countable $\mathcal{M} \models T, \mathrm{Aut}(M) \curvearrowright M$ is oligomorphic;
- for some countable $\mathcal{M} \models T, \mathrm{Aut}(M) \curvearrowright M$ is oligomorphic.

EXERCISE 3. Let $\mathcal{M}$ be an infinite $\mathcal{L}$-structure and $A = \{a_1, \ldots, a_n\} \subset M$. Show that $\mathrm{Th}(M; a_1, \ldots a_n)$ is ω-categorical if and only if $\mathrm{Th}(M)$ is ω-categorical.

Let $A \subset M$ be finite. Show that X is an A-definable subset of M^k if and only if X is an $\mathrm{Aut}(M/A)$-invariant subset of M^k, where by $\mathrm{Aut}(M/A)$ we denote the automorphisms of M fixing A.

Definition 2. Let $A \subseteq M$. We say that $b \in M$ is **algebraic** over A if there is a finite A-definable subset $X \subseteq M$ such that $b \in X$.

We write $\mathrm{acl}(A)$ for the union of finite A-definable subsets of M.

EXERCISE 4. Suppose that M is a model of an ω-categorical theory. Show that for each finite $A \subset M$, $\mathrm{acl}(A)$ is finite. Deduce that there is no ω-categorical theory extending the theory of infinite fields.