Extra exercises are marked with a $\star\star$. I DO NOT EXPECT YOU TO ANSWER THEM. I hope they can bring you joy.

In the following exercises we work in an $\mathcal{L}$-theory T with $|\mathcal{L}| \leq \kappa$.

EXERCISE 1. Prove that if $\mathcal{M}$ is κ-saturated then it is κ-homogeneous.

EXERCISE 2. Show that if $\mathcal{M}$ is κ-saturated, then it is κ^+-universal.

EXERCISE 3. Show that if $\mathcal{M}$ is $|M|$-homogeneous, then it is strongly $|M|$-homogeneous.

EXERCISE 4. Show that $\mathcal{M}$ is κ-saturated if and only if it is κ-homogeneous and κ^+-universal.

EXERCISE 5. Prove the following lemma from the lecture: let X be a definable subset of $\mathbb{M}$ and A a set of parameters (i.e. a set of size $< \kappa$ inside of $\mathbb{M}$). Then, the following are equivalent:

(a) X is definable over A;

(b) X is $\mathrm{Aut}(\mathbb{M}/A)$-invariant (i.e. invariant under automorphisms of $\mathbb{M}$ fixing A pointwise).

If you want to work with monster models without moving out of ZFC, the following extra exercise will allow you to be at ease with your set theoretic conscience.

Definition 1. An infinite structure $\mathcal{M}$ of cardinality κ is **special** if it is the union of an elementary chain $(\mathcal{M}_\lambda)_{\lambda<\kappa}$, where the λ are *cardinals* of size $< \kappa$ and each $\mathcal{M}_\lambda$ is κ^+-saturated.

$\star\star$ **EXERCISE 6.** Show that the following hold:

(a) If $\mathcal{M}$ is saturated then it is special;

(b) A special structure of regular cardinality is saturated;

(c) Suppose that $\lambda < \nu$ implies $2^\lambda \leq \nu$. Then, T has a special model of cardinality ν;

(d) A special structure of cardinality κ is κ^+-universal and strongly $\mathrm{cf}(\kappa)$-homogeneous.