Exercises with a $\star\star$ are extra and I do not expect you to do them.

EXERCISE 1. Let $p \in S_x(M)$ be a definable type. Show that for each $B \supseteq M$, p has a unique extension $q \in S_x(B)$ which is definable over M.

EXERCISE 2. Let $\phi(x,y)$ be stable and let $p \in S_\phi(M)$ for M a model. Show that p is definable by a Boolean combination of ϕ^{opp}-formulas with parameters from M. [Hint: imitate the proof of Erdös-Makkai, which you proved in a previous problem sheet. In particular, note that for all finite $c_0, \dots, c_n \in M$, p is not defined by a Boolean combination of ϕ^{opp}-formulas with parameters from the c_i. Hence, you can inductively build sequences $(b_i)_{i<\omega}, (b_i)'_{i<\omega}$, and $(c_j)_{j<\omega}$ such that either $(b_i c_i)_{i<\omega}$ or $(b'_i c_i)_{i<\omega}$ yield that ϕ has the order property.]

$\star\star$ **EXERCISE 3.** Show that in the exercise above you can choose the Boolean combination of ϕ^{opp}-formulas to be positive. [Hint: the proof is almost identical but you need to be slightly more careful in the construction of the sequences. In particular, keep in mind that $X \subsetneq A$ is a positive Boolean combination of $X_0, \dots, x_n$ if and only if for all $x, y \in A$ if $x \in X$ and, for every $i \leq n$, we have that if $x \in X_i$, then $y \in X_i$, then $y \in X$.]

Definition 1. Let X be a topological space. The **derived set** X' is the set of limit points of X (i.e. the set of non-isolated points). For an ordinal, we define inductively the **Cantor-Bendixon derivative** $X^{(\alpha)}$: $X^{(0)} = X$, $X^{(\alpha^+)} := \left(X^{(\alpha)}\right)'$, and for λ a limit ordinal, $X^{(\lambda)} := \bigcap_{\alpha<\lambda} X^{(\alpha)}$. We will need this only for finite ordinals.

EXERCISE 4. Let $\phi(x,y)$ be stable. Let $X \subseteq S_\phi(\mathbb{M})$ be closed and non-empty. Then, $X^{(n+1)} = \emptyset$ for some $n \geq 0$. Moreover, (choosing n to be minimal such that $X^{(n+1)} = \emptyset$), $X^{(n)}$ is finite. [Hint: show that if $X^{(n+1)} \neq \emptyset$, there is a binary tree of parameters $(b_s | s \in < n+12)$ witnessing the (finite) binary tree property of height $n+1$. For finiteness, first observe that every $X^{(i)}$ is closed.]