EXERCISE 1. Let $\mathcal{M}$ be a model and $p \in S_\phi(M)$. Show that p extends uniquely to a generalised ϕ-type.

Definition 1. Let $\phi(x, y)$ be an $\mathcal{L}$-formula. Let $A \subseteq \mathbb{M}$. We denote by $\mathrm{FER}_\phi(A)$ the collection of equivalence relations $E(x, y)$ on $\mathbb{M}$ with finitely many classes such that for each $a \in \mathbb{M}$ the equivalence class of a, $E(\mathbb{M}, a)$ is equivalent to a Boolean combination of ϕ-formulas over A.

Theorem 2 (Finite Equivalence Relations Theorem)**.** *Let $\phi(x, y)$ be a stable. Let p be a generalised ϕ-type over $A \subseteq \mathbb{M}$. Let*

$$Y := \{q(x) \in S_\phi(\mathbb{M}) | q(x) \textit{ is an extension of } p \textit{ definable over } \mathrm{acl}^{eq}(A)\}.$$

Then, Y is finite, $\mathrm{Aut}(\mathbb{M}/A)$ *acts transitively on Y, and there is an equivalence relation $E \in$* $\mathrm{FER}_\phi(A)$ *such that for all $q_1, q_2 \in Y$, $q_1 = q_2$ if and only if $q_1(x) \cup q_2(y) \vdash E(x, y)$.*

EXERCISE 2. Prove the Finite Equivalence Relations Theorem.

Definition 3. Let X be an $\emptyset$-definable subset of $\mathbb{M}$. We say that X is **stably embedded** if every $\mathcal{L}(M)$-definable $Y \subseteq X$ is $\mathcal{L}(X)$-definable (i.e. definable already with parameters from X).

EXERCISE 3. Let X be an $\emptyset$-definable subset of $\mathbb{M}$. Show that the following are equivalent:

- X is stably embedded;
- Every type $\mathrm{tp}(a/X)$ is definable over some $C \subseteq X$;
- For every a there is a small (i.e. of size $< |\mathbb{M}|$) subset $C \subseteq X$ such that $\mathrm{tp}(a/C) \vdash \mathrm{tp}(a/X)$;
- Every automorphism of X extends to an automorphism of $\mathbb{M}$.

EXERCISE 4. Let T be stable. Let X be an $\emptyset$-definable subset of $\mathbb{M}$. Show that X is stably embedded.