

EXERCISE 1. Consider $\text{Th}(\mathbb{Q}, \text{cyc})$, where

$$\mathbb{Q} \models \text{cyc}(a, b, c) \Leftrightarrow (a < b < c) \vee (b < c < a) \vee (c < a < b).$$

Show that for $a \neq b$, $\text{cyc}(a, x, b)$ divides over \emptyset . Show that $x = x$ forks over \emptyset , but does not divide over \emptyset .

EXERCISE 2. Let $\phi(x, y)$ be stable. Show that $\phi(x, b)$ does not divide over A if and only if it is satisfiable in every model containing A .

Now, consider a stable theory T . Let $p \in S_x(B)$ and let $A \subseteq B$. Show that the following are equivalent:

- p does not fork over A ;
- there is a global type extending p which is $\text{acl}^{eq}(A)$ -invariant;
- there is a global type extending p which is $\text{acl}^{eq}(A)$ -definable.

EXERCISE 3. Show the following are equivalent:

- $\text{tp}(a/Ab)$ does not divide over A ;
- For every infinite A -indiscernible sequence I such that $b \in I$, there is some $a' \equiv_{Ab} a$ such that I is Aa' -indiscernible;
- For every infinite A -indiscernible sequence I such that $b \in I$ there is some $J \equiv_{Ab} I$ such that J is Aa -indiscernible.

EXERCISE 4. Use the above exercise to show the following property of dividing:

Suppose $\text{tp}(a/B)$ does not divide over $A \subseteq B$ and $\text{tp}(b/Ba)$ does not divide over Aa . Then, $\text{tp}(ab/B)$ does not divide over A .

EXERCISE 5. Let Δ be as above. Show that there is $\psi_\Delta(x, y_0, \dots, y_n, z, z_0, \dots, z_{2n})$ such that

- if $|A| \geq 2$, each Δ -formula over A is equivalent to some $\psi_\Delta(x, \bar{a})$ for \bar{a} a tuple from A ;
- any consistent formula of the form $\psi_\Delta(x, \bar{a})$ for \bar{a} a tuple from A is equivalent to a Δ -formula over A ;
- if all formulas in Δ are stable, then so is ψ_Δ .