ZOMBIE EXERCISE 1. Consider a stable theory T. Let $p \in S_x(B)$ and let $A \subseteq B$. Show that the following are equivalent:

- p does not fork over A;
- there is a global type extending p which is $\operatorname{acl}^{eq}(A)$-invariant;
- there is a global type extending p which is $\operatorname{acl}^{eq}(A)$-definable.

Theorem 1 (Independence in Stable Theories). *Let T be stable. Then, $\mathop{\smile\!\!\!\!\!\!|}\,$ (considered as a ternary relation on small subsets of $\mathbb{M}$) satisfies the following properties:*

1. *(Invariance) If $A \mathop{\smile\!\!\!\!\!\!|}_C B$ and $\sigma \in \operatorname{Aut}(\mathbb{M})$, then $\sigma A \mathop{\smile\!\!\!\!\!\!|}_{\sigma C} \sigma B$;*
2. *(Finite Character) $A \mathop{\smile\!\!\!\!\!\!|}_C B$ if and only if $A_0 \mathop{\smile\!\!\!\!\!\!|}_C B_0$ for all finite $A_0 \subseteq A, B_0 \subseteq B$;*
3. *(Extension) if $A \mathop{\smile\!\!\!\!\!\!|}_C B$, then for any D there is $A' \equiv_{CB} A$ such that $A' \mathop{\smile\!\!\!\!\!\!|}_C BD$;*
4. *(Algebraicty) If $A \mathop{\smile\!\!\!\!\!\!|}_C A$, then $A \subseteq \operatorname{acl}^{eq}(C)$;*
5. *(Local Character) For A finite and for any B there is $C \subseteq B$ such that $|C| \leq |T|$ and $A \mathop{\smile\!\!\!\!\!\!|}_C B$;*
6. *(Transitivity and Monotonicity) $A \mathop{\smile\!\!\!\!\!\!|}_C B$ and $A \mathop{\smile\!\!\!\!\!\!|}_{CB} D$ if and only if $A \mathop{\smile\!\!\!\!\!\!|}_C BD$ (with transitivity being the $\Rightarrow$ implication and transitivity being the $\Leftarrow$ implication);*
7. *(Symmetry) $A \mathop{\smile\!\!\!\!\!\!|}_C B$ if and only if $B \mathop{\smile\!\!\!\!\!\!|}_C A$;*
8. *(Stationarity) Let a and b be finite. Suppose that $a \equiv_{\operatorname{acl}^{eq}(C)} b$ and $a \mathop{\smile\!\!\!\!\!\!|}_C D$ and $b \mathop{\smile\!\!\!\!\!\!|}_C D$. Then, $a \equiv_{\operatorname{acl}^{eq}(AC)} b$.*

EXERCISE 2. Prove Theorem 1.[See Lecture notes for hint with references to previous theorems.]