

**EXERCISE 1.** Let  $T$  be an arbitrary theory. Suppose that  $p \in S(\mathbb{M})$  forks over  $A$  with  $|A| \leq \kappa$ . Then  $p$  has  $\geq \kappa$  many  $\text{Aut}(\mathbb{M}/A)$ -conjugates.

**EXERCISE 2.** Let  $T$  be stable. Show there is an ordinal  $\alpha$  such that if  $U(p) \geq \alpha$ , then  $U(p) = \infty$ . [Hint: use local character.]

**Theorem 1** (Equivalents to superstability). *Let  $T$  be stable. The following are equivalent:*

1.  $T$  is superstable (in the sense of Definition ??);
2. Every type has ordinal-valued  $U$ -rank;
3. For every finite tuple  $a$  and every  $B$  there is some finite  $C \subseteq B$  such that  $a \downarrow_C B$ .

**EXERCISE 3.** Prove the implication (3)  $\Rightarrow$  (1) in Theorem 1.

**ZOMBIE EXERCISE 4.** Consider a stable theory  $T$ . Let  $p \in S_x(B)$  and let  $A \subseteq B$ . Show that the following are equivalent:

- $p$  does not fork over  $A$ ;
- there is a global type extending  $p$  which is  $\text{acl}^{eq}(A)$ -invariant;
- there is a global type extending  $p$  which is  $\text{acl}^{eq}(A)$ -definable.