EXERCISE 1. Prove the following: let $\mathcal{M}$ be ω-saturated. Suppose that $\phi \in \mathcal{L}(M)$ is minimal in $\mathcal{M}$. Then ϕ is strongly minimal.

EXERCISE 2. Prove Neumann's Lemma: Let $A, B \subseteq \mathbb{M}$ and $(c_1, \ldots, c_n)$ a sequence of elements not algebraic over A. Show that $\operatorname{tp}(c_1, \ldots, c_n / A)$ has a realisation which is disjoint from B.

EXERCISE 3. Show that $\operatorname{acl}(A)$ is the intersection of all models containing A.

EXERCISE 4.

(a) Consider the theory of $(\mathbb{Z}, s)$, the integers with the successor operation $s(x) = x + 1$. This theory has quantifier elimination. What is algebraic closure in this theory? Is this $x = x$ in $(\mathbb{Z}, s)$ minimal? is it strongly minimal?

(b) Consider the theory of $(\mathbb{N}, <)$. This theory has quantifier elimination if we add a function symbol for the successor and a constant symbol for 0 (both of which are definable in the original theory). Is $x = x$ in $(\mathbb{N}, <)$ minimal? is it strongly minimal?

Definition 1. A set of definable subsets of $\mathbb{M}$ in the variable x, $I \subseteq \operatorname{Def}_x(\mathbb{M})$ is an **ideal** if it contains $\emptyset$, and it is closed under (definable) subsets and finite unions.

EXERCISE 5. Prove the following:
Let $I \subseteq \operatorname{Def}_x(\mathbb{M})$ be an ideal. Let $\pi(x)$ be a partial type over A (closed under conjunctions) such that $p(\mathbb{M})$ is not contained in any set in I. Then, for every $B \supseteq A$, there is a type $q \in S(B)$ extending p and such that $q(\mathbb{M})$ is not contained in any set in I.