EXERCISE 1. Let $f : A \to B$ be an elementary bijection between sets of parameters. Then, f extends to an elementary bijection $f' : \mathrm{acl}(A) \to \mathrm{acl}(B)$.

EXERCISE 2. Let T be a strongly minimal theory (not necessarily countable). Show the following:

(a) Every infinite algebraically closed set of parameters S is the universe of a model of T;

(b) A model $\mathcal{M}$ is ω-saturated if and only if $\dim(M) \geq \aleph_0$;

(c) All models are ω-homogeneous.

Definition 1. Let A be a set of parameters and x a tuple of variables. For an $\mathcal{L}(A)$-formula $\phi(x)$, we define

$$[\phi(x)] := \{p \in S_x(A) | \phi(x) \in p\}.$$

Sets of the form $[\phi(x)]$ form a basis of clopen sets for a topology on $S_x(A)$, which we call the **Stone topology**.
We say that a type $p \in S_x(A)$ is **isolated** if there is some $\mathcal{L}(A)$-formula $\psi(x)$ such that $[\psi(x)] = \{p\}$.

Fact 2. *The type space $S_x(A)$ with the Stone topology is compact, Hausdorff, and totally disconnected (i.e. for all $p, q \in S_x(A)$ there is a clopen set X such that $p \in X$ and $q \notin X$.*

EXERCISE 3. Let T be a countable complete theory. Let A be a countable set of parameters and x a finite tuple of variables. Suppose that $|S_x(A)| < 2^{\aleph_0}$. Prove the following:

- the isolated types in $S_n(A)$ are dense, i.e. for any $\mathcal{L}(A)$-formula $\phi(x)$, $[\phi]$ contains an isolated type;
- $|S_x(A)| \leq \aleph_0$.

[Hint: in both contexts, you need to build an adequate binary tree of $\mathcal{L}(A)$-formulas $(\phi_\sigma | \sigma \in 2^{<\omega})$ such that any finite branch is consistent but any two children of a common node are mutually inconsistent. Then, each infinite branch of the binary tree can be used to construct a type, giving $2^{\aleph_0}$-many.]