Next time I will also ask to present some of the exercises on modules from the previous problem sheet.

Corollary 1. *For $E_1, \dots E_k \in \mathrm{ER}(T)$ and $\phi(x_1^{E_1}, \dots x_k^{E_k})$ an $\mathcal{L}^{eq}$-formula, there is an $\mathcal{L}$-formula $\psi(\overline{y}_1, \dots \overline{y}_k)$ such that*

$$T^{eq} \vdash \forall \overline{y}_1 \dots \overline{y}_k (\psi(\overline{y}_1 \dots \overline{y}_k) \leftrightarrow \phi(\pi_{E_1}(\overline{y}_1) \dots \pi_{E_k}(\overline{y}_k))).$$

EXERCISE 1. Let F be the forgetful map

$$F : S_{(S_=)^n}(T^{eq}) \to S_n(T)$$

sending types of real n-tuples in T^{eq} to their restriction to an n-type in T. Show that F is a homeomorphism. Prove Corollary 1.

EXERCISE 2. Let κ be an infinite cardinal with $\kappa \geq |\mathcal{L}|$. Show the following:

- If $\mathcal{M}$ is κ-saturated, then $\mathcal{M}^{eq}$ is κ-saturated;
- If $\mathcal{M}$ is strongly κ-homogeneous, then $\mathcal{M}^{eq}$ is strongly κ-homogeneous;
- If T is κ-categorical, then T^{eq} is κ-categorical.
- If T is κ-stable, then T^{eq} is κ-stable;
- If T is stable, then T^{eq} is stable;
- If T is NIP, then T^{eq} is NIP;
- If T is NSOP, then T^{eq} is NSOP.

For X a non-empty set and $G \curvearrowright X$ we can endow G with the pointwise-convergence topology, where stabilizers of finite sets G_A for $A \subseteq X$ finite form a basis of clopen neighbourhoods of the identity. Hence, the cosets of stabilizers of finite sets gG_A for $A \subseteq X$ finite and $g \in G$ form a basis of clopen sets.

EXERCISE 3. Consider $\mathrm{Aut}(M) \curvearrowright M$ for M countable and ω-categorical. Note that $\mathrm{Aut}(M) = \mathrm{Aut}(M^{eq})$. Prove that the open subgroups of $\mathrm{Aut}(M)$ are precisely the stabilizers of imaginaries $\mathrm{Aut}(M/e)$ for $e \in M^{eq}$. [Hint: the ($\Leftarrow$) direction does not use ω-categoricity. For the ($\Rightarrow$) direction, note that for any $H \leq \mathrm{Aut}(M)$ and $\overline{a} \in \mathrm{Aut}(M)$ the equivalence relation E on the $\mathrm{Aut}(M)$-orbit of $\overline{a}$ $\mathrm{Orb}(\overline{a})$ given by

$$E(g_1\overline{a}, g_2\overline{a}) \text{ if and only if } g_2^{-1}g_1 \in H$$

is a 0-definable relation.]