Next time I will also ask to present some of the exercises on modules from the previous problem sheet.

EXERCISE 1. Let X be a definable subset of $\mathbb{M}^n$. Let A be a set of parameters, and let b be a canonical parameter for X.

- Show that X is definable over A if and only if $b \in \mathrm{dcl}^{eq}(A)$;
- Show that the following are equivalent:
  - X is **almost A definable**, i.e. there is an A-definable equivalence relation E on n-tuples with finitely many classes and such that X is a union of E-classes;
  - $\{\sigma(X) | \sigma \in \mathrm{Aut}(\mathbb{M}/A)\}$ is finite;
  - $\{\sigma(X) | \sigma \in \mathrm{Aut}(\mathbb{M}/A)\} < |\mathbb{M}|$;
  - $b \in \mathrm{acl}^{eq}(A)$.

EXERCISE 2. Show that T has EI if and only if for each $\phi(\overline{x}) \in \mathcal{L}(\mathbb{M})$ there is $\psi(x, y) \in \mathcal{L}$ such that there is a unique $\overline{a} \in \mathbb{M}$ such that $\phi(\mathbb{M}) = \psi(\mathbb{M}, \overline{a})$.

EXERCISE 3. Show that T^{eq} eliminates imaginaries. [Hint/thoughts: techically, there is no problem in defining imaginaries for a multi-sorted structure and so check the original definition. However, it is probably more convenient to show that definable subsets of $\mathbb{M}^{eq}$ have canonical parameters in $\mathbb{M}^{eq}$.]

EXERCISE 4. Show that the following are equivalent

1. T has weak elimination of imaginaries;
2. For every definable X there is a finite set $\{\overline{a}_1, \ldots, \overline{a}_n\}$ such that for all $\sigma \in \mathrm{Aut}(\mathbb{M})$
$$\sigma(X) = X \text{ if and only if } \sigma \text{ permutes the } \overline{a}_i;$$
3. For each $\phi(\overline{x}) \in \mathcal{L}(\mathbb{M})$ there is $\psi(\overline{x}, \overline{y})$ such that there is only a finite number of $\overline{a}$ such that $\phi(\mathbb{M}) = \psi(\mathbb{M}, \overline{a})$;
4. For every definable X there is a smallest (real) algebraically closed set A such that X is A-definable.

EXERCISE 5. Let $\mathcal{M}$ be the countable model of an ω-categorical theory. The following are equivalent:

- $\mathrm{Th}(M)$ has WEI;
- for all $A, B \subseteq M$ algebraically closed, we have
$$\mathrm{Aut}(M/A \cap B) = \langle \mathrm{Aut}(M/A), \mathrm{Aut}(M/B) \rangle.$$